Merit Factors of Complex Polyphase Sequences Based on

A Real Measure Of Aperiodic Autocorrelation

Preecha Kocharoen* • Termpong Srited**

Abstract

Barker sequences are well known with good aperiodic autocorrelation, i.e. Binary Barker sequence of length $L = 13$ $\{a_n\} = (0000011001010)$. With the property that in-phase autocorrelation function is maximum and out-of-phase aperiodic autocorrelation peak equal or less than 1. It is desirable, in radar systems and many other communication applications, to make the out-of-phase aperiodic autocorrelation function as small as possible. This paper investigates the binary, polyphase ternary, and polyphase quaternary sequences with good correlation properties, which satisfy the condition of Barker constraint and have the minimum out-of-phase aperiodic autocorrelation peak by using the real-part polyphase method.

Index Terms *Aperiodic autocorrelation, Complex-valued polyphase sequences, Barker Sequences, Merit Factor.*

INTRODUCTION

Sequences with good aperiodic correlation properties are extremely useful in digital systems and communications engineering. It is desirable to make the out-of-phase aperiodic auto-correlation values as small as possible and the aperiodic autocorrelation function (ACF) merit factor as great as possible [1].

Because the aperiodic correlation properties of a sequence are very difficult to analyze, only a few analytical treatments have been published. The main method of investigation is on an empirical basis via computer search algorithms.

In this paper, computer searching has been employed to find the best possible Binary sequences up to length $L = 32$, Polyphase Ternary up to $L = 26$, and Polyphase Quaternary up to $L = 24$.

This paper will introduce the new method on real-part polyphase ternary and quaternary sequences, which calculate only the real part of the correlation instead of considering the complex-valued correlation.

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BINARY APERIODIC CORRELATION FUNCTION

There are two types of the aperiodic correlation function, *aperiodic autocorrelation function and aperiodic crosscorrelation function.* The aperiodic crosscorrelation function between two sequences $\{a_r\} = (a_o\ a_1\ a_2 \ldots a_{L-1})$ and $\{b_r\} = (b_o\ b_1\ b_2 \ldots b_{L-1})$ of length L is defined as

$$C_{a,b}(\tau) = \begin{cases} \sum_{r=0}^{L-1-\tau} \hat{a}_r . \hat{a}_{r+\tau}, & 0 \le \tau \le L-1 \\ \sum_{r=0}^{L-1+\tau} \hat{a}_{r-\tau} . \hat{b}_r, & 1-L \le \tau < 0 \\ 0 & |\tau| \ge L \end{cases} \qquad (1)$$

where $\{a_r\}$ and $\{b_r\}$ are represented in +1 and -1 form.

Note that the sequences here are of finite length L and are not necessarily single periods of periodic sequences of L. Eqn 1 can be defined the aperiodic autocorrelation, when $a = b$, and rewritten as :

$$C_a(\tau) = \sum_{r=0}^{L-1-\tau} \hat{a}_r . \hat{a}_{r+\tau}, \quad 0 \le \tau \le L-1 \qquad (2)$$

In addition, the aperiodic autocorrelation function and aperiodic crosscorrelation function can be defined in unnormalized mod-2 form as :

$$C_a(\tau) = \sum_{r=0}^{L-1-\tau} a_r \oplus a_{r+\tau}, \quad 0 \le \tau \le L-1 \qquad (3)$$

and;

$$C_{a,b}(\tau) = \begin{cases} \sum_{r=0}^{L-1-\tau} a_r \oplus b_{r+\tau}, & 0 \le \tau \le L-1 \\ \sum_{r=0}^{L-1+\tau} a_{r-\tau} \oplus b_r, & 1-L \le \tau < 0 \\ 0 & |\tau| \ge L \end{cases} \qquad (4)$$

where $\{a_r\}$ and $\{b_r\}$ are represented in 0 and 1 mod-2 form.

The aperiodic autocorrelation also can be defined using the simple measure of agreements and disagreements as

$$C_a(\tau) = A_\tau - D_\tau, \qquad (5)$$

where A_τ is the number of agreements and D_τ is the number of disagreements between the signal sequence $\{a_r\}$ and its shift by τ places.

Similarly, the aperiodic crosscorrelation also can defined in simple measure agree and disagree form as :

$$C_{ab}(\tau) = A_\tau - D_\tau, \qquad (6)$$

where A_τ is the number of agreements and D_τ is the number of disagreements between $\{a_r\}$ and a shift of $\{b_r\}$ by τ places.

Figure 1 illustrates the aperiodic cross-correlation of sequence A and sequence B of period L.

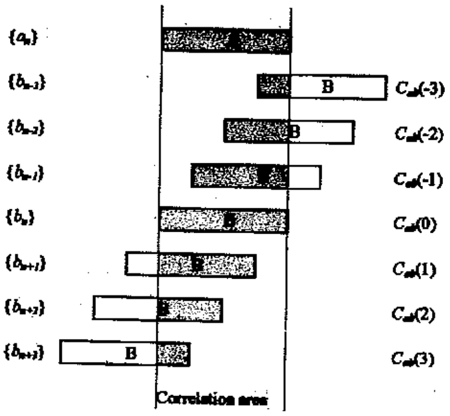

Figure 1. The aperiodic crosscorrelation

The maximum nontrivial value of aperiodic correlation $C_{\max}$ is defined by

$$C_{max} = max\{C_{am}, C_{cm}\} \qquad (7)$$

where C_{am} is the maximum out-of-phase autocorrelation value, and C_{cm} is the maximum crosscorrelation value.

The Merit Factor (MF) is an indicator, which involves the ratio of the energy of correlation function mainlobes to the energy of the correlation sidelobes. For the autocorrelation function (ACF), the mainlobe occurs at zero shift, $C_a(0)$. The merit factor can be written in the following equation :

$$MF = \frac{C_a(0)^2}{2.\sum_{\tau=1}^{L-1}|C_a(\tau)|^2} \qquad (8)$$

In radar and the other communication applications aspire to make the nontrivial aperiodic autocorrelation peak as small as possible and the aperiodic ACF merit factor as great as possible. In addition, it is also desirable to make the nontrivial aperiodic crosscorrelation peak as small as possible.

BARKER SEQUENCES

Let $\{a_r\} = (a_o\ a_1\ a_2 \ldots a_{L-1})$ be a complex-valued sequence and $\{a^*_r\}$ be its complex conjugate, it can be defined the aperiodic ACF as :

$$C_a(\tau) = \sum_{r=0}^{L-1-\tau} \hat{a}_r.\hat{a}^*_{r+\tau}, \quad 0 \le \tau \le L-1 \qquad (9)$$

The sequences fall into the Barker sequences category if their aperiodic ACF sidelobes are equal or less than 1,

$$|C_a(\tau)| \le 1 \qquad (10)$$

Binary Barker Sequences

Barker sequences have very good merit factor, which is desirable in many communication applications, such as radar systems and spread-spectrum communication systems. So far, only 7 Binary Barker sequences have been found, as listed in Table 1. In particular, the binary Barker sequence of length $N = 13$ has the highest known merit factor of 14.08.

Table 1. *Binary Barker Sequences*

L	Binary Barker Sequence $\{a_n\}$	MF
2	00	2.00
3	001	4.50
4	0001	4.00
5	00010	6.25
7	0001101	8.17
11	00011101101	12.10
13	0000011001010	14.08

COMPLEX MEASURE ON POLYPHASE SEQUENCES

Using the complex qth roots of unity as unit vectors spaced uniformly round the unit circle in the complex plane can represent the components of polyphase sequences[1,3,4,5,6]. Thus, this polyphase representation gives rise to q complex quantities of the form $e^{j2\pi k/q}$ where $0 \le k \le q-1$, mutually displaced by an angle of $2\pi/q$. Figure 2 shows an example of a complex cube roots of unity, which q = 3 the angle between the vectors is $2\pi/3$.

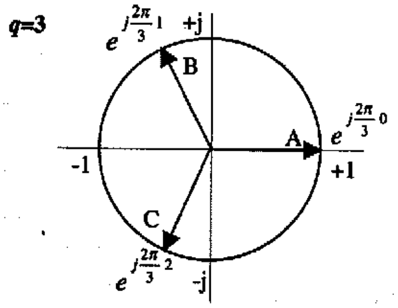

Figure 2. the complex cube roots of unity

Let $z = e^{j2\pi/q}$, so the elements become $z^0 = 1, z^1, z^2, \ldots, z^{q-1}$ and multiplication on these values is equivalent to adding their exponents $\bmod -q$. i.e. $z^r . z^s = z^t$ where $t = r \oplus s \bmod -q$. Also, the complex conjugate of z^r is z^{-r}, so that multiplying by the complex conjugate is equivalent to subtracting the exponents $\bmod -q$. $i.e. z^r = z^{-s} = z^t$ where $t = r \ominus s \bmod -q$. It follows that multiplication on the roots of unity is a closed operation. Consequently, a convenient way of representing these polyphase sequences is provided by listing the sequence of integers mod-q which form the exponents of z.

The definition of the aperiodic correlation between two complex-valued sequences $a = \{\hat{a}_0 \hat{a}_1 \ldots \hat{a}_{L-1}\}$ and $b = \{\hat{b}_0 \hat{b}_1 \ldots \hat{b}_{L-1}\}$ of length L, where $\hat{a}_i = z^{a_i}$ and b_i $\hat{b}_i = z^{b_i}$, with a_i and taken as integers $\bmod -q$, at a relative shift of τ places can be written as :

$$C_{ab}(\tau) = \sum_{r=0}^{L-\tau-1} \hat{a}_r . \hat{b}_{r+\tau} = \sum_{r=0}^{L-\tau-1} a_r \Theta b_{r+\tau} \qquad (11)$$

where $\hat{b}_i^*$ is the complex conjugate of $\hat{b}_i$. For aperiodic autocorrelation we have

$$C_a(\tau) = \sum_{r=0}^{L-\tau-1} \hat{a}_r . \hat{a}_{r+\tau}^* \qquad (12)$$

As multiplication of the roots of unity, and addition and subtraction mod-q, are closed operations, the correlation values are made up from sums of the roots of unity. In both cases, the correlation values are the sums of the individual correlations between the complex-valued components of a and b. The results of probably come out in real values, when a and b are in-phase, in antiphase, or orthogonal, or come out in complex values. If the results are in complex, taking modulus of this quantity is the way to represent the correlation value.

A REAL MEASURE POLYPHASE SEQUENCES

A. Concepts of a Real Measure Polyphase Sequences

Polyphase sequences can be represented

by using the complex qth roots of unity as unit vectors equally displaced round the unit circle in the complex plane. Each qth roots have its own q complex quantities of the form $e^{j2\pi k/q}$, where $0 \le k \le q-1$, mutually displaced by an angle of $2\pi/q$. An example of the complex plane for Binary, Polyphase Ternary, and Polyphase Quaternary sequences can be shown in Figure 3, 4 and 5.

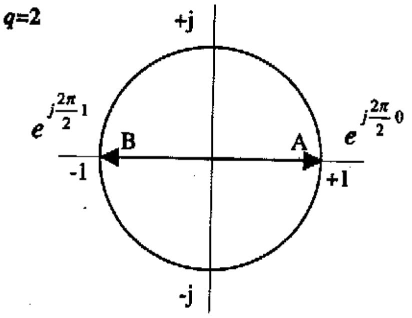

Figure 3. Complex plane of a. Binary Sequence

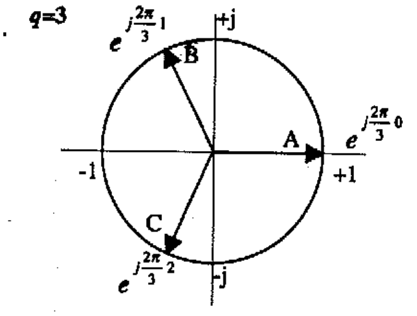

Figure 4. Complex plane of a. Ternary Sequence

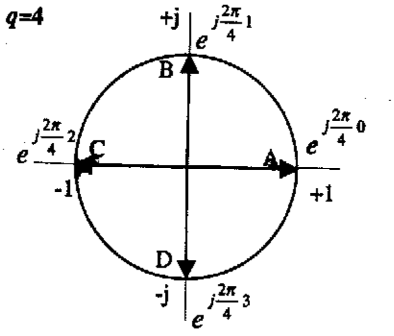

Figure 5. Complex plane of a. Polyphase Quaternary Sequence

The correlation function of the complex polyphase sequences may contain complex values, which are dealt with by taking their modulus. On the other hand, the correlation functions of the real measure polyphase sequences are based on the cosine of the phase angle between components of the complex polyphase sequences.

In this measure, the correlation between a sequence $\{a_r\} = (a_0\, a_1\, a_2 \ldots a_{L-1})$ of length L and its shift by τ places obtains from taking only the real part of the sum of the products $a_r . a^*_{r+\tau}$ rather than its modulus. If $\varnothing_{r,r+\tau}$ is the angle between the phases represented by the sequence elements a_r and $a_{r+\tau}$, then the definition of the aperiodic autocorrelation of a polyphase sequence $\{a_r\}$ can be expressed as :

$$C(\tau) = \sum_{r=0}^{L-\tau-1} \cos\left(\varnothing_{r,r+\tau}\right) \qquad (13)$$

Consider the possible situations depicted in Figure 6. Here $\theta = 2\pi k/q$ and $\varnothing$ is some multiple of θ. Thus, a_r and b_s both represent qth roots of unity and $\varnothing$ is the angle between them. In Figure 6a, a_r and b_s are in-phase so that $\varnothing = 0$, also $\cos\varnothing$ = 1. In Figure 6b, a_r and b_s are in antiphase so that $\varnothing = \pi$, also $\cos\varnothing$ = -1. In Figure 6c, a_r and b_s are orthogonal so that $\varnothing = \pi/2$, also $\cos\varnothing$= 0. In Figure 6d, b_s leads a_r by $\varnothing$, now the correlation can be represented in cosine of the angle between a_r and b_s. The result will come out only the real value.

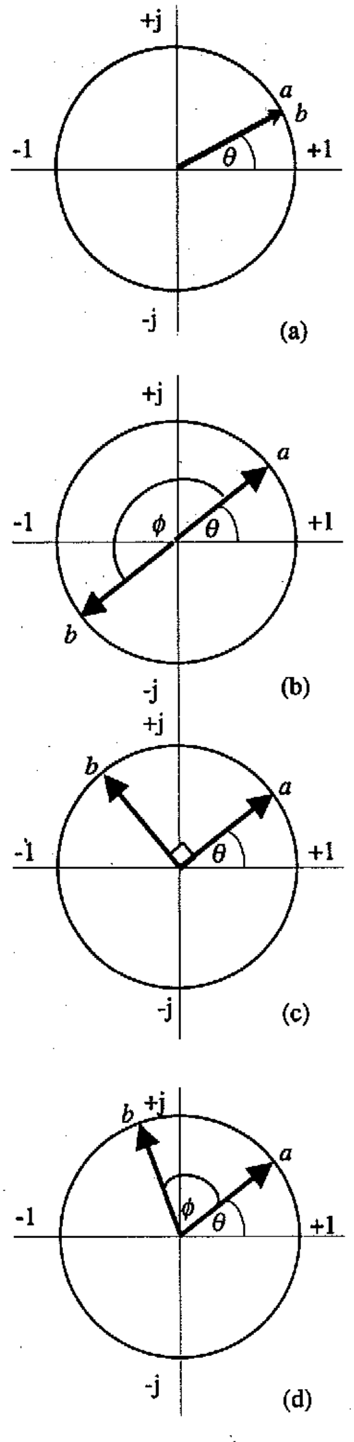

Figure 6. Correlation between polyphase sequence symbols.
a. in-phase, ∅ = 0 *b. in antiphase, ∅ = π*
c. orthogonal, ∅ = π / 2. *d. phase angle ∅*

B. Binary Sequences

In the case $q=2$, the square roots of unity, +1 and -1, are both real and the angle ∅ between them is π, so cos ∅ = +1 or -1. By using the real measure method with Binary sequences, it gives the same results as the methods that have been mentioned in [1,2]. Therefore, the highest known merit factor remains 14.08 of the Binary Barker sequence length $L=13$.

Taking the autocorrelation sidelobe into account can discover the maximum merit factor that can possible be reached the upper bound. In this binary case, each value of correlation, $c(\tau)$ is formed from a sum of +1s and -1s and so it can easily deduce the minimum autocorrelation sidelobe values. When the autocorrelation function of the sequences have the even length overlap, this may yield an equal number of +1s and -1s so that $c(\tau)$ = 0 on these occasions. On the other hand, if the overlap has odd length, the correlation process will always not balance. The minimum values of correlation becomes $c(\tau)=\pm 1$. For the complete autocorrelation spectrum, τ will vary from $-(L-1)$ to $+(L-1)$ and so the out-of-phase overlap will vary from 1 to $L-1$. For even L, there will be $L/2$ occasions when the overlap is odd, and for odd L the opportunity of the odd overlap will be $(L-1)/2$. By using Eqn 8, the upper bound merit factor of binary sequences can be rewritten as :

$$MF_{ub} = \begin{cases} L & for L even \\ \dfrac{L^2}{L-1} & for L odd \end{cases} and q = 2 \qquad (14)$$

Figure 7 shows the upper bound merit factor of binary sequences up to length $L = 32$.

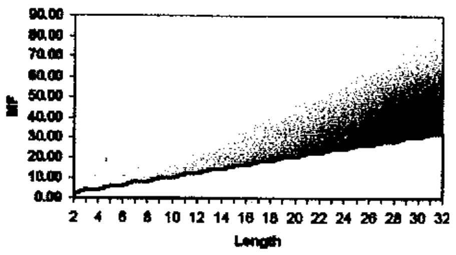

Figure 7. The upper bound merit factor of binary sequences up to $L = 32$

C. Polyphase Ternary Sequences

The complex cube roots of unity are given by $z^k = e^{j2\pi k/3}$, with k = 0, 1, or 2. Therefore, $z^0 = e^0 = +1, z^1 = e^{j2\pi/3} = \left(-1 + j\sqrt{3}\right)/2$ and $z^2 = e^{i\pi/3} = \left(-1 + j\sqrt{3}\right)/2$, these three unit vectors uniformly displaced by $2\pi/3 = 120^{\circ}$. The possible angles between phase components of three phase sequences are shown in Table 2 and their real measure, $\cos\varnothing$, also shown in Table 3.

Table 2. *The possible angle between phase components of three phase sequences*

$\varnothing$	A	B	C
A	0	$2\pi/3$	$2\pi/3$
B	$2\pi/3$	0	$2\pi/3$
C	$2\pi/3$	$2\pi/3$	0

Table 3. *The real measure between phase components of three phase sequences*

$Cos\oplus$	A	B	C
A	+1	-1/2	-1/2
B	-1/2	+1	-1/2
C	-1/2	-1/2	+1

This implies that if, during correlations, two symbols agree their correlation is +1 and if they disagree their correlation is $-\frac{1}{2}$. Thus, the correlation values in the real measure polyphase ternary case is given by :

$$C(\tau) = A_\tau - \tfrac{1}{2} D_\tau \qquad (15)$$

where A_τ is the number of agreements and D_τ is the number of disagreements between the sequences and its shift by τ places. *i.e.* the overlap is $L - \tau$ digits.

Now, for an overlap of one digit the minimum correlation value can be $-\frac{1}{2}$. For an overlap of two digits, the minimum correlation value can be $+\frac{1}{2}$. i.e. one agreement and one disagreement $C(\tau) = 1 - \frac{1}{2} - \frac{1}{2} = 0$. When an overlap of three digits in which there is one agreement and two disagreements, a minimum value for $C(\tau) = 1 - \frac{1}{2} - \frac{1}{2} = 0$. The pattern of minimum values is repeated as the overlap runs from 1 to $L-1$, so that the upper bound on the merit factor for ternary sequences can be written as

$$MF_{ub} = \begin{cases} 3L & for L \equiv 0 \bmod -3 \\ \dfrac{3L^2}{L-1} & for \equiv 1 \bmod -3 \quad for q \equiv 3 \\ \dfrac{6L^2}{2L-1} for L \equiv 2 \bmod -3 \end{cases} \qquad (16)$$

Figure 8 shows the upper bound merit factor of polyphase ternary sequences up to Length $L = 26$.

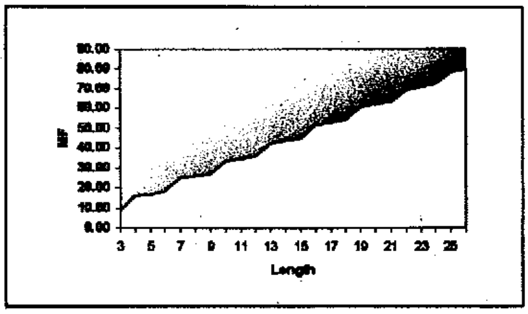

Figure 8. The upper bound merit factor of polyphase ternary sequences up to L=26

D. Polyphase Quaternary Sequences

The complex fourth roots of unity are given by $z^k = e^{j2\pi k/4}$, with k = 0, 1, 2, or 3 and these four unit vectors mutually displaced by $\pi/2 = 90^\circ$. The possible angles between phase components of four phase sequences are shown in Table 4 and their $\cos\varnothing$ also shown in Table 5.

Table 4. The possible angle between phase components of four phase sequences

Ø	A	B	C	D
A	0	$\pi/2$	π	$\pi/2$
B	$\pi/2$	0	$\pi/2$	π
C	π	$\pi/2$	0	$\pi/2$
D	$\pi/2$	π	$\pi/2$	0

Table 5 : The real measure between phase components of four phase sequences

$\cos\varnothing$	A	B	C	D
A	+1	0	-1	0
B	0	+1	0	+1
C	-1	0	+1	0
D	0	-1	0	+1

From Table 5, the minimum correlation value can possible be zero at all out-of-phase positions and would consequently exhibit infinity merit factors.

EXAMPLES OF COMPUTING THE APERIODIC CORRELATION OF POLYPHASE TERNARY AND POLYPHASE QUATERNARY SEQUENCES.

Example 1 Compute the polyphase ternary aperiodic autocorrelation function of the sequence $\{a_n\} = (000102)$.

$$\{a_n\}: \; 0 \; 0 \; 0 \; 1 \; 0 \; 2$$
$$\{a_n\}: \; \underline{0 \; 0 \; 0 \; 1 \; 0 \; 2}$$
$$\varnothing: \; 0 \; 0 \; 0 \; 0 \; 0 \; 0$$

$$\cos\varnothing: \; 1 \; 1 \; 1 \; 1 \; 1 \; 1 \qquad C_a(0) = 6$$

$$\{a_n\}: \; 0 \; 0 \; 0 \; 1 \; 0 \; 2$$
$$\{a_n + 1\}: \; \underline{0 \; 0 \; 1 \; 0 \; 2}$$
$$\varnothing: \; 0 \; 0 \; 2\pi/3 \; 2\pi/3 \; 2\pi/3$$

$$\cos\varnothing: \; 1 \; 1 \; -1/2 \; -1/2 \; -1/2 \qquad C_a(1) = +1/2$$

$$\{a_n\}: \; 0 \; 0 \; 0 \; 1 \; 0 \; 2$$
$$\{a_n + 3\}: \; \underline{1 \; 0 \; 2}$$
$$\varnothing: \; 2\pi/3 \; 0 \; 2\pi/3$$

$$\cos\varnothing: \; -1/2 \; 1 \; -1/2 \qquad C_a(3) = 0$$

$$\{a_n\}: \; 0 \; 0 \; 0 \; 1 \; 0 \; 2$$
$$\{a_n + 4\}: \; \underline{0 \; 2}$$
$$\varnothing: \; 0 \; 2\pi/3$$

$$\cos\varnothing: \; 1 \; -1/2 \qquad C_a(4) = +1/2$$

$\{a_n\}$: 0 0 0 1 0 2
$\{a_n+5\}$: 2
$\varnothing$: $2\pi/3$

$\cos\varnothing$: $-1/2$ $\qquad C_a(5) = -1/2$

The aperiodic autocorrelation $\{C_a(\tau)\} = (6, +1/2, +1, 0, +^{+}1/2, -1/2)$

The merit factor :

$$MF = \frac{6^2}{2.\left\{1/2^2 + 1^2 + 0^2 + 1/2^2 + (-1/2)^2\right\}} = 10.29$$

Example 2 Compute the polyphase quaternary aperiodic autocorrelation function of the sequence $\{a_n\} = (000213)$.

$\{a_n\}$: 0 0 0 2 1 3
$\{a_n\}$: 0 0 0 2 1 3
$\varnothing$: 0 0 0 0 0 0

$\cos\varnothing$: 1 1 1 1 1 1 $\qquad C_a(0) = 6$

$\{a_n\}$: 0 0 0 2 1 3
$\{a_n+1\}$: 0 0 2 1 3
$\varnothing$: 0 0 π $\pi/2$ π

$\cos\varnothing$: 1 1 -1 0 -1 $\qquad C_a(1) = 0$

$\{a_n\}$: 0 0 0 2 1 3
$\{a_n+2\}$: 0 2 1 3
$\varnothing$: 0 π $\pi/2$ $\pi/2$

$\cos\varnothing$: 1 -1 0 0 $\qquad C_a(2) = 0$

$\{a_n\}$: 0 0 0 2 1 3
$\{a_n+3\}$: 2 1 3
$\varnothing$: π $\pi/2$ $\pi/2$

$\cos\varnothing$: -1 0 0 $\qquad C_a(3) = 0$

$\{a_n\}$: 0 0 0 2 1 3
$\{a_n+4\}$: 1 3
$\varnothing$: $\pi/2$ $\pi/2$

$\cos\varnothing$: 0 0 $\qquad C_a(4) = 0$

$\{a_n\}$: 0 0 0 2 1 3
$\{a_n+5\}$: 3
$\varnothing$: $\pi/2$

$\cos\varnothing$: 0 $\qquad C_a(5) = 0$

The aperiodic autocorrelation $\{C_a(\tau)\} = (6, 0, 0, -1, 0, 0)$

The merit factor :

$$MF = \frac{6^2}{2.\left\{0^2 + 0^2 + (-1)^2 + 0^2 + 0^2\right\}} = 18.00$$

BEST POSSIBLE SEQUENCES BASED ON COMPUTER SEARCH ALGORITHM

A. Best possible Binary Sequences

By using the computer program search the best possible binary sequences. So far, the best possible binary sequences up to length $N = 32$, as shown in Table 6 have been found.

For binary, the sequence of length $N = 13$ has the highest known merit factor of 14.08 and its autocorrelation function $\{C(\tau)\} = (13,0,1,0,1,0,1,0,1,0,1,0,1)$.

Figure 9 shows graph of **Upper bound Merit factor against The Optimum Merit factor** of binary sequences. The upper bound shows the highest merit factor which possible to reach. On the contrary, the Optimum shows the highest found merit factor.

Table 6 : Best Possible Binary Sequences Up To Length $N = 32$

Length N	Number	Peak SL C_{am}	Merit F MF	Example Sequence for Length N $\{a_n\}$
2	1	1	2.00	01
3	1	1	4.50	001
4	2	1	4.00	0001
5	1	1	6.25	00010
6	7	2	2.57	000010
7	1	1	8.17	0001101
8	4	2	4.00	00001101
9	6	3	3.38	000011010
10	10	2	3.85	0000011010
11	1	1	12.10	00011101101
12	4	2	7.20	000001100101
13	1	1	14.08	0000011001010
14	18	2	5.16	00000011001010
15	2	3	7.50	000001100110101
16	8	3	5.33	0000001110011010
17	11	2	4.52	00001100100101011
18	4	2	6.48	000001011010001100
19	2	3	6.22	0000101011110011011
20	2	3	7.69	00000101110100111001
21	1	3	8.48	001111111001101010110
22	6	3	6.21	0000001110010101001101
23	6	3	5.63	00000000111001010110010
24	2	3	8.00	001100011111010101101110
25	2	3	8.68	0001110000000101011011001
26	4	3	7.51	00000011100111010101101101
27	1	3	9.85	000111100010001000100101101
28	2	2	7.84	0001111000100010001001011011
29	2	3	6.78	00011000111111010101101110010
30	4	4	7.63	000001111101101101010101110011 10
31	1	3	7.17	0000000111000110010011010100101
32	1	4	8.00	00000001010010101110001100100110

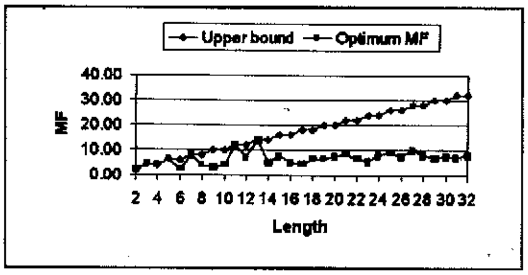

Figure 9. Upper bound MF against the Optimum MF of binary sequences

B. Best possible Polyphase Ternary and Polyphase Quaternary Sequences

Up to now, the best possible polyphase ternary sequences up to length $N = 26$ and the best possible polyphase quaternary sequences up to length $N = 24$, have been found as shown in Table 7 and Table 8.

Polyphase ternary, only the sequences of length 25 are not the Barker sequences, the maximum autocorrelation function sidelobe C_{am} are 1.5. However, their merit factor are very high, 33.78, higher than the ordinary ternary sequence, which are not more than 18.06[1].

The polyphase quaternary sequences of length $N = 4, 8, 10, 16,$ and 20 have the infinity value merit factor, the maximum autocorrelation function sidelobe C_{am} are 0. In addition, the autocorrelation function at zero shift are equal the values of their length. Up to length $N = 4$, only the even-length sequences have been found that the sequences reach the maximum ACF merit factor. Moreover, the odd-length sequences have been found that the maximum merit factor only reach,

$$MF = \frac{N^2}{N-1} \qquad (17)$$

Furthermore all the best possible polyphase quaternary sequences are under the Barker sequences constraint, $|C_a(\tau)| \le 1$.

Figure 10, and 11 show graphs of *Upper bound Merit factor* against the Optimum Merit factor of polyphase ternary, and polyphase quaternary sequences. The upper bound shows the highest merit factor, which possible to reach on the other hand, the Optimum shows the highest found merit factor.

Table 7: Best Possible Polyphase Ternary Sequences Up To Length $N = 26$

Length N	Number	Peak SL C_{am}	Merit F MF	Example Sequence for Length N $\{a_n\}$
3	1	0.5	9.00	001
4	2	1.0	6.40	0012
5	2	0.5	16.67	00102
6	7	1.0	10.29	000102
7	2	0.5	24.50	0001202
8	2	0.5	25.60	00110102
9	2	0.5	27.00	001100202
10	1	1.0	22.22	0001210110
11	2	1.0	24.20	00012101102
12	2	0.5	36.00	000012201202
13	11	1.0	24.14	0001110120102
14	1	1.0	32.67	01010011211002
15	3	1.0	34.62	000010122011021
16	4	1.0	26.95	0001212011011102
17	3	1.0	28.90	00011011022010202
18	4	1.0	30.86	001111001202102010
19	1	1.0	30.08	0000012202012021102
20	4	1.0	32.00	00000122020120211021
21	4	1.0	33.92	000012120222012202102
22	2	1.0	33.38	0010212111110021102012
23	2	1.0	35.27	00100222220122002121020
24	2	1.0	37.16	000011110210102212011201
25	1	1.5	33.78	0010012101200001220202211
26	2	1.0	32.98	00001212011120102110122022

Table 8: Best Possible Polyphase Quaternary Sequences Up To Length $N = 24$

Length N	Number	Peak SL C_{am}	Merit F MF	Example Sequence for Length N $\{a_n\}$
4	2	0	Infinity	0013
5	7	1	6.25	00132
6	2	1	18.00	000213
7	15	1	8.17	0000213
8	12	0	Infinity	00021131
9	707	1	10.12	000012031
10	8	0	Infinity	0011030231
11	33888	1	12.10	00000220131
12	20	1	72.00	000020211331
13	6456	1	14.08	0000012301313
14	32	1	49.00	00002132011201
15	16574	1	16.07	000001202032113
16	96	0	Infinity	0000022011331313
17	Over 30000	1	18.06	00000022011331313
18	40	1	54.00	000110330220113131
19	Over 30000	1	20.06	0000002102331102313
20	68	0	Infinity	00000202201133111313
21	Over 30000	1	22.05	000000122002023110313
22	?	1	121.00	0000011230321032020313
23	Over 30000	1	24.05	00000002220020123013131
24	7	1	72.00	000000222001230230231313

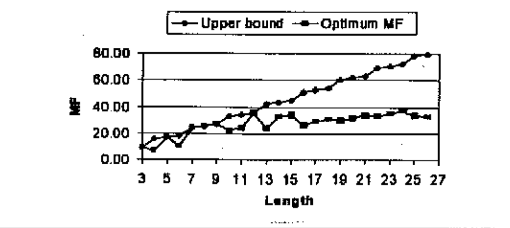

Figure 10. Upper bound MF against the Optimum MF of Polyphase Ternary sequences

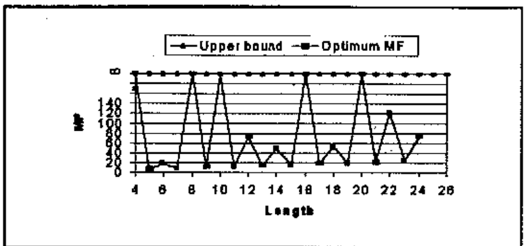

Figure 11. Upper bound MF against the Optimum MF of Polyphase Quaternary sequences

CONCLUSIONS

This paper has investigated the consequences of employing a real measure of correlation on complex-valued polyphase sequences, rather than the conventional complex form of derivation.

The optimum Binary sequences up to length $N = 32$ were found by complete search algorithm. There are only 7 Binary Barker sequences have been found and these sequences reach the upper bound.

The Polyphase Ternary sequences with the optimum merit factor were found by exhaustive search up to length $N = 26$. So far, only 7 lengths of the sequences are achieved the upper bound and the longest known sequence that gets to the upper bound has only 12 digits. Most of the sequences that have been found have peak sidelobes equal or less than 1 and qualify the Barker constraint, $C(\tau) \leq 1$.

The Polyphase Quaternary sequences with the optimum merit factor up to length $N = 24$ were found by complete search. There are 5 values of N, all even, which reach the upper bound. The longest known sequence that has an infinite merit factor has 20 digits. ❑

References

1 Fan, P. and Darnell, M. **Sequence Design for Communications Applications.** Research Studies Press, John-Wiley, Taunton, 1996.

2 Lindner, J. **Binary sequences up to length 40 with best possible autocorrelation function,** Electron. Lett., 1975, 11, (21), p.507.

3 Chang, N. and Golomb, S.W. **On n-phase Barker Sequences,** IEEE. Trans. Inf. Theory, 1994, 40, (4), pp.1251-1253.

4 Golomb, S.W. and Scholtz, R.A. **Generalized Barker sequences,** IEEE Trans. Inf. Theory, 1965, IT-11,(4), pp.533-537.

5 Bmer, L. and Antweiler, M. **Polyphase Barker sequences,** Electron. Lett., 1989, 25, (23), pp. 1577-1579.

6 Friese, M. and Zottmann, H. **Polyphase Barker sequences up to length 31,** Electron. Lett., 1994, 30, (23), pp.1930-1931.